Drag-reducing air-water interfaces according to immersed biological objects

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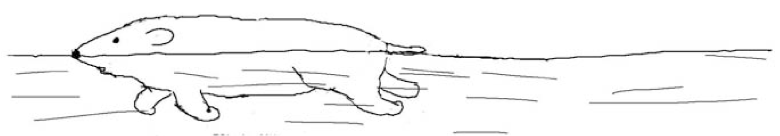
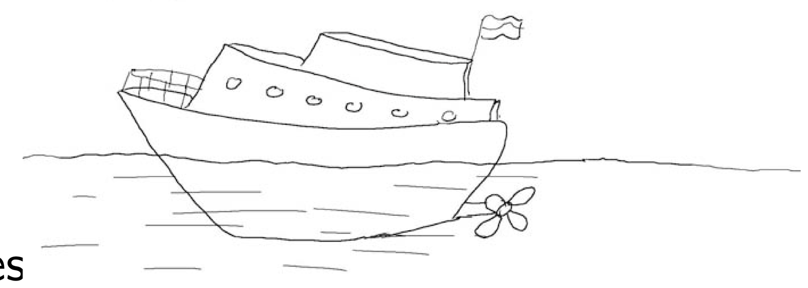
1. Objectives

- Analysis of (drag-reducing) air layers around biological objects:
  Existense & Shape,
  Persistence,
  Stability

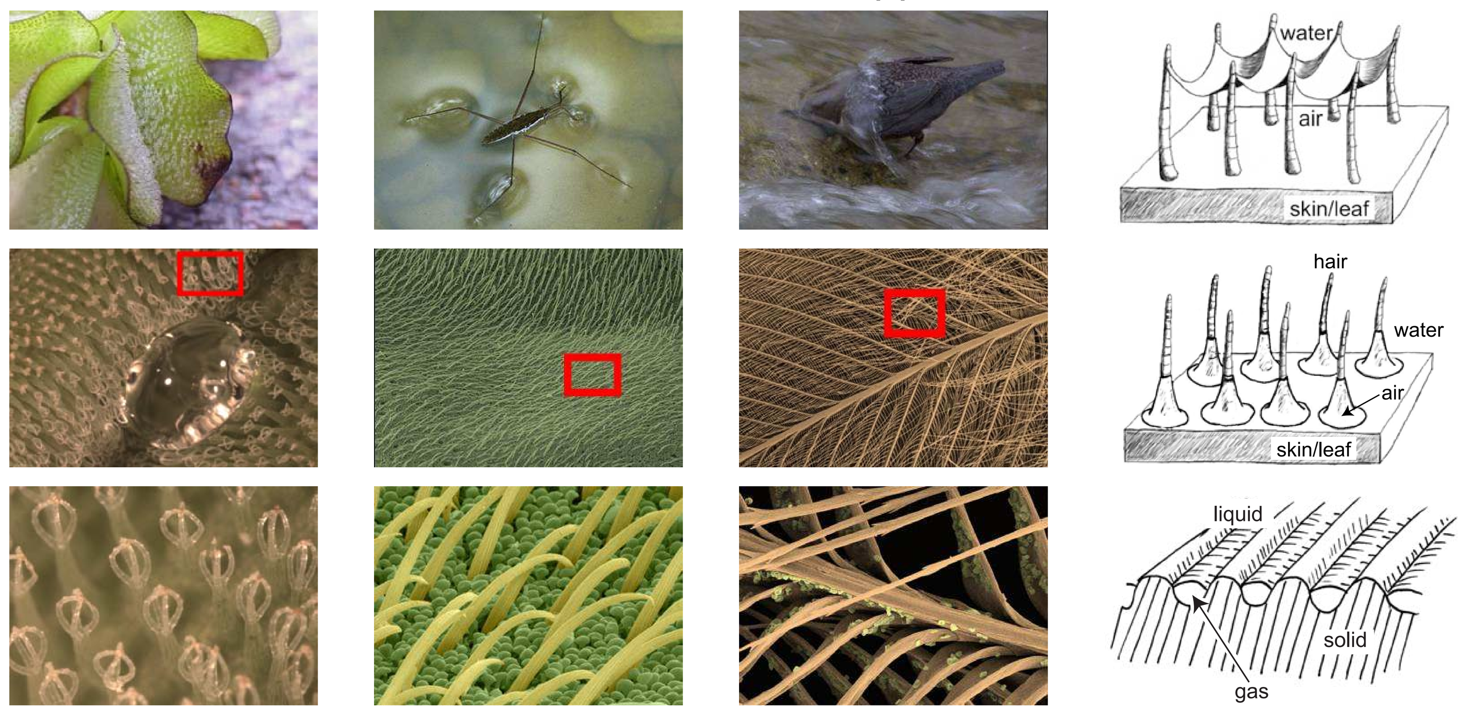


- Transfer of results to drag-reducing technical surfaces
- Side Effects: Antifouling, Minimizing corrosion, "Dry" textiles



2. Biological models

Salvinia | Water strider | Dipper



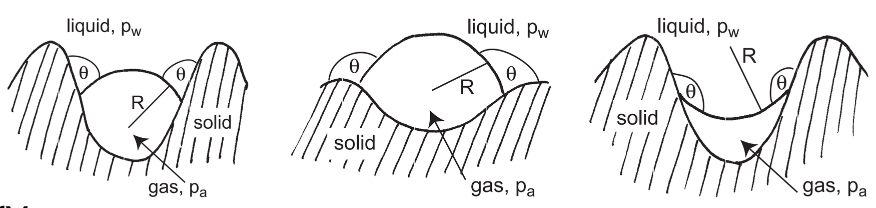
Photos: A. Roth-Nebelsick, Tübingen and Nees-Institut, Bonn
Drawings: B. Binder, Tübingen

3. Physical basis of interfaces

Existence & shape

Young-Laplace-Equation, contact angle θ, solid geometry

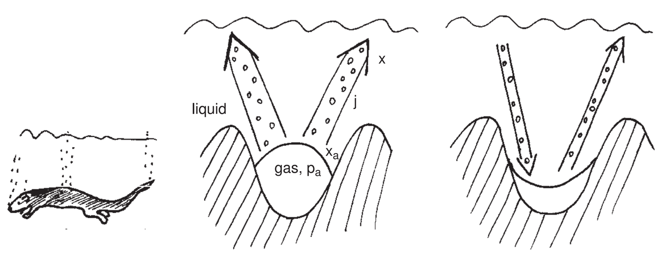
$$p_a - p_w = \pm\sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$



Persistence

Laws of Henry and Fick

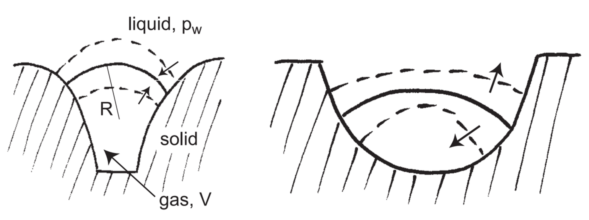
$$p_a = x_a\, k_H \qquad \mathbf{j} = -D_a\, grad(x)$$



Stability

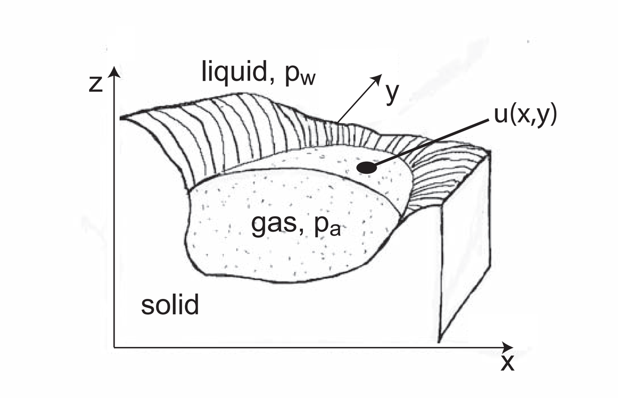
Form of solid surface

$$\xi = \left[\frac{1}{V}\left(p_w + \frac{2\sigma}{R}\right) - \frac{2\sigma}{R^2}\frac{dR}{dV}\right]_0$$



4. Young-Laplace-Equation

$$p_a - p_w = \pm\,\sigma\,\frac{\left(1 + {u_x}^2\right)u_{yy} - 2u_x u_y u_{xy} + \left(1 + {u_y}^2\right)u_{xx}}{\left(1 + {u_x}^2 + {u_y}^2\right)^{3/2}}$$

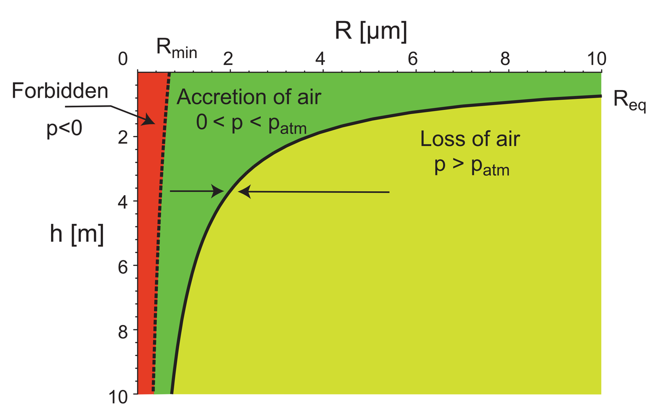


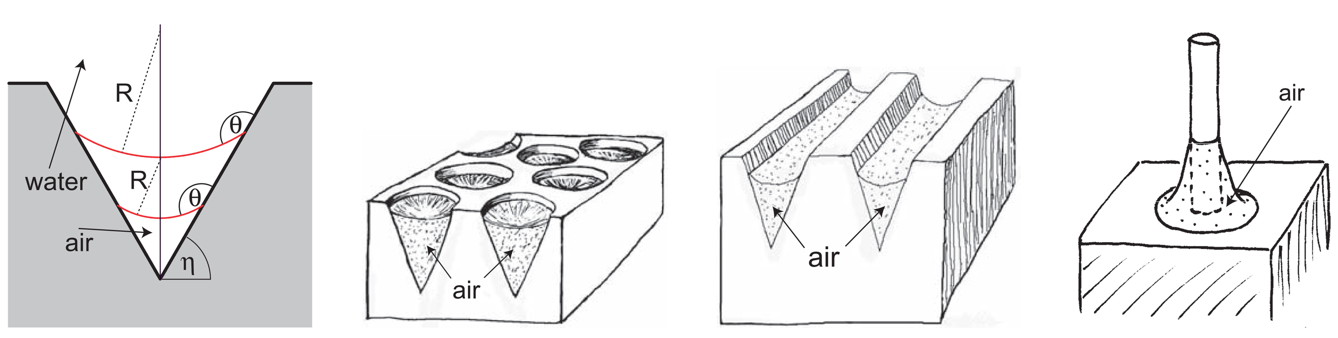
Solution procedures:

1. Derivation of mathematical (non-)existence theorems.
   → Apeltauer, Pedit (Tübingen)
2. Equivalent formulation as a variational problem, application of computational brute-force methods.
   → Frauendiener (Otago)
3. Adaptation of known solutions to compatible boundary conditions.
   → Roth-Nebelsick, Konrad (Tübingen)

5. First results

- Highly symmetric air layers over matching solids can be investigated analytically.
- Air layers over V-shaped solids *exist*, *persist* and are *stable*, provided
  (i) contact angle θ is appropriate
  (ii) $R_{min} := \frac{\sigma}{\rho g h + p_{atm}} < R \leq \frac{\sigma}{\rho g h} =: R_{eq}$





6. Future

Identification of air-water interfaces which

- minimise the contact area between water and solid (⟹ minimise surface friction)
- allow a wide and varying range of contact angles
- allow solid structures greater than a few micrometers
- do not collapse (mechanical stability)
- do not drain away (persistence)