Strain

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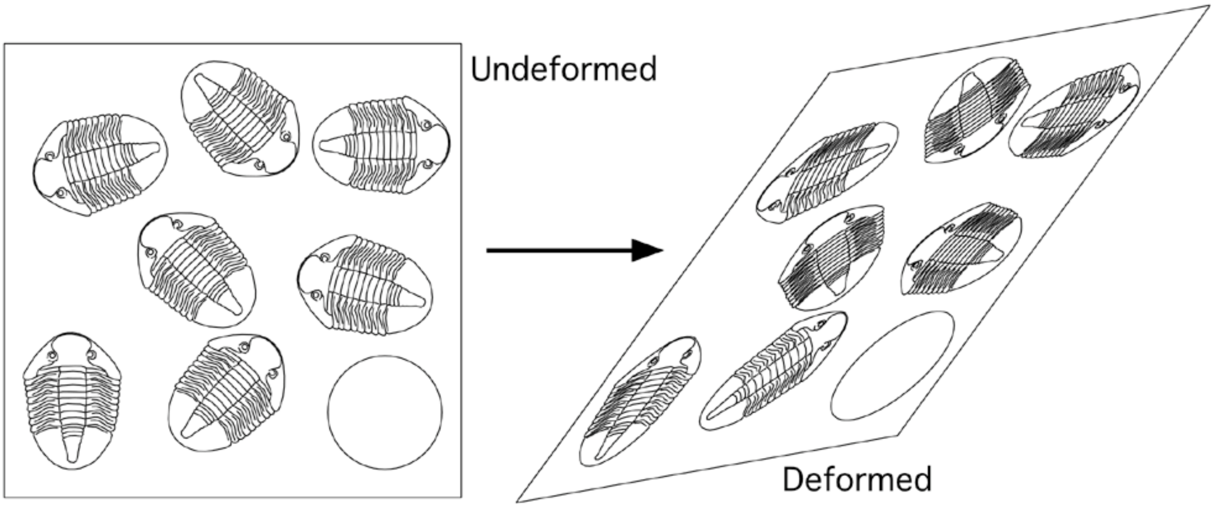

Figure 1. Population of Asaphus trilobites before and after a homogeneous deformation event. The amount of strain is visualised by a circle in the undeformed state, which has been deformed to the strain ellipse in the deformed state. Note that all deformed trilobites look different, although they all experienced exactly the same amount of deformation.

Strain

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Strain is the change of shape resulting from stresses acting on a volume of material. Deformation involves four components

1. Rigid body translation
2. Rigid body rotation
3. Relative rotation of material lines with respect to each other
4. Stretching and shortening of material lines

All of these four occur in the example of figure 1. The first two do not produce a change of shape of an object and are normally difficult to determine. The trilobites may have been incorporated on the shelf in front of one continent, which then drifted and rotated across the globe. This would already constitute a rigid body translation and rotation. So, when we speak about strain, we essentially only speak of the last two components of deformation.

Homogeneous strain

Strain normally varies from point to point within a rock. However, we can also normally define a small area within the heterogeneous strain field, where we can approximately say that strain is homogeneous. *Homogeneous strain* in a volume means that all material points within that volume experienced the same amount of strain. Below, we look at the description of homogeneous strain, and will see that one can also use a Mohr circle construction here.

Finite strain

Producing strain takes time, typically hundreds of thousands to millions of years in geology. We of course always see the end product of deformation in the rock record. Our task is to figure out what happened: we analyse the strain that we observe in the rock and try to quantify it. This way we can reconstruct the situation before deformation and in general get more insight into the events that produced the strain.

Finite strain
Incremental strain
Strain rate

A period of straining produces a certain amount of strain at the end, which we call the *finite strain*. The finite strain is the sum of many small increments of strain. The amount of strain accumulated during a small increment of time is the *incremental strain*. The *strain rate* is the rate of strain accumulation and has unit [s^{-1}].

The position gradient tensor

Figure 2 shows a general case of homogeneous deformation of a small volume of material. The position of material points in the undeformed state can be described by a vector ($\underline{p}$). After deformation, each particle will have a new position in space, which can also be represented by a vector ($\underline{q}$). As we already know, we can use a tensor to convert one vector into another. The same holds for position vectors during strain. If we do not have any rigid body translation, the particle in the origin of our reference system remains there: $\underline{p}$ = (0,0) goes to $\underline{q}$ = (0,0).

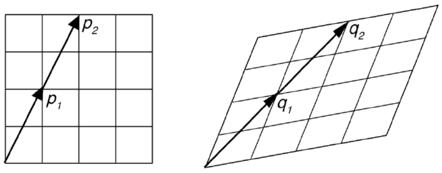


Figure 2. Example of homogeneous deformation. Two vectors $\underline{p}_1$ and $\underline{p}_2$ deform to different vectors $\underline{q}_1$ and $\underline{q}_2$. The figure shows that in homogeneous deformation any straight line remains a straight line.

If the above holds, i.e. we do not take rigid body translation into account, we can describe the relationship between $\underline{p}$ and $\underline{q}$ with a tensor multiplication:

$$q_i = \mathbf{F}_{ij} p_j \tag{1}$$

Written out in full this means that we have 2 equations (in 2 dimensions):

$$\begin{aligned} q_x &= \mathrm{F}_{xx} p_x + \mathrm{F}_{xy} p_y \\ q_y &= \mathrm{F}_{yx} p_x + \mathrm{F}_{yy} p_y \end{aligned} \tag{2}$$

Position gradient tensor

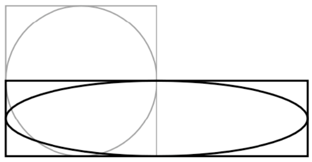
The tensor $\mathbf{F}_{ij}$ is called the *position gradient tensor*, which converts the position of a material point in the undeformed state to the one in the deformed state.

Main types of strain

There are four main types of strain, which each have their particular type of position gradient tensor. These are end-member types, meaning that in reality one usually has a combination of two or more of these types. The four types are described below for the 2-dimensional case:

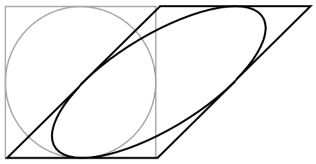
Pure shear

Pure shear: $\mathbf{F}_{ij} = \begin{pmatrix} 2 & 0 \\ 0 & 1/2 \end{pmatrix}$



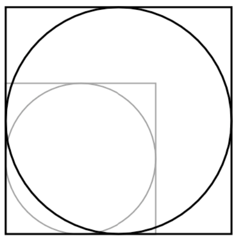
Simple shear

Simple shear: $\mathbf{F}_{ij} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$



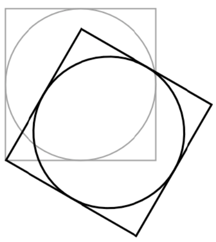
Area change (dilation)

Area change: $\mathbf{F}_{ij} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$



Rotation

Rotation: $\mathbf{F}_{ij} = \begin{pmatrix} \cos(30) & \sin(30) \\ -\sin(30) & \cos(30) \end{pmatrix}$



Stretching, shortening and rotation of lines

Extension

Strain causes a change of length of lines. We can define the *extension* (e) of a line which had length l_0 before deformation and l_1 after deformation with:

$$e = \frac{l_1 - l_0}{l_0} \quad (3)$$

A line that does not change its length has e=1, a stretching line has e>1 and a shortening line has e<1.

Rotation

Strain also causes a *rotation* ω of lines, which is the difference between the orientation of a line in the undeformed and deformed state.

Finite stretching axes

Finite strain ellipse

Finite stretching axes

Finite strain ratio

A circle subjected to homogeneous strain ***always*** deforms to an ellipse (or a circle, which is also an ellipse). This ellipse we call the *finite strain ellipse*. In 3 dimensions the equivalent is the *strain ellipsoid*. As any ellipse, the strain ellipse has principal axes, perpendicular to each other. These axes represent the maximum and minimum extensions: e_1 and e_2 respectively in 2 dimensions. These axes we call the *finite stretching axes* (FSAs).

The ratio of the maximum and minimum FSA we call the *finite strain ratio* (*Rf* or *Rs*), which is of course always equal or greater than one. In three dimensions we can measure two ratios of FSAs: e_1/e_2 and e_2/e_3. The ratio (k) between these two ratios again depends the shape of the finite strain ellipsoid. Here we can have three cases:

- $k > 1$ ($e_1/e_2 > e_2/e_3$): The strain ellipsoid is *prolate* (cigar-shaped)
- $k < 1$ ($e_1/e_2 < e_2/e_3$): The strain ellipsoid is *oblate* (pancake-shaped)
- $k = 1$ ($e_1/e_2 = e_2/e_3$): The strain ellipsoid is neither prolate nor oblate.

Plane strain

If we have no volume change during deformation and there is zero stretching in the direction of the intermediate FSA (e_2=1) we must have that $e_3=1/e_1$, which means that k=1. This case is called *plane strain*.

Strain from passively deformed objects

In an ideal world, all rocks would have spherical markers embedded in them. With those one could then determine the finite strain of each deformed rock. As always, the world is not ideal and we normally do not have such perfect markers. However, some rocks do have objects in them that can be used as strain markers. Some examples are:

- originally round (sand) grains,
- round worm burrows (*Scolithus*),
- reduction spots in shales,
- mafic enclaves in granitoids,
- ooids, etc.

Under the assumption that these objects deformed just as much as the whole rock, we can use their deformed shape to determine the amount of strain. If the objects were originally spheres (like ooids), the method is straightforward: the finite elliptical shape is the finite strain ellipse. In many cases however, the objects were not spheres to start with. At the most one can assume they were originally roughly elliptical.

Consider the population of elliptical objects in figure 2a. They have various shapes and orientations. After deformation (Fig. 2b) they have changed their shapes, but none of the objects has exactly the shape of the strain ellipse. It is then more difficult to determine the true amount of strain.

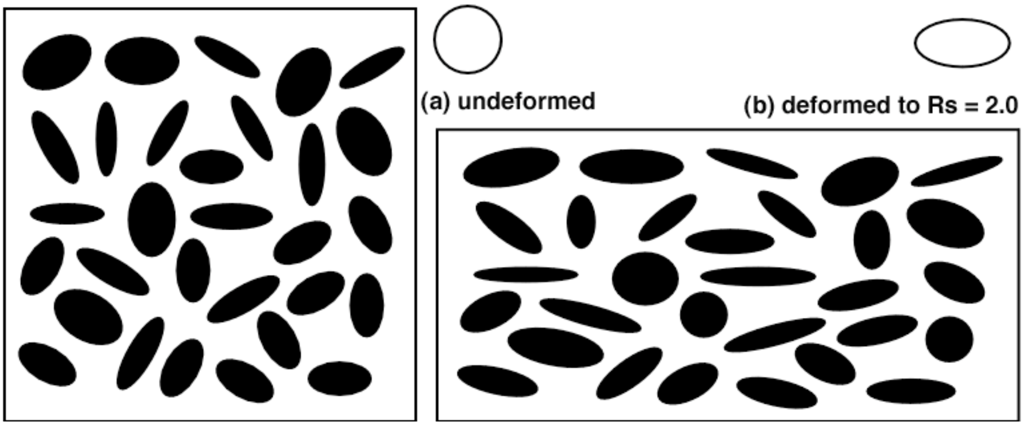
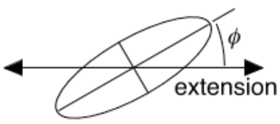


Figure 2. Population of elliptical objects with various shapes and orientations. **(a)** Undeformed state. **(b)** Deformed state. White ellipse shows the amount of strain (Rs=2.0).

The *Rf*-ϕ method

The *Rf*-ϕ method is used for populations of originally approximately elliptical objects, such as mafic enclaves in granitoids. The method yields the axial ratio (*Rs*) of the finite strain ellipse and the orientation of that ellipse. It cannot be used to determine any volume or area changes as it only gives the shape of the strain ellipse.



The method is based on the idea that an object with an initial ellipticity (*Ri*) and orientation (ϕi) deforms together with the host rock to a different ellipticity (*Rf*) and orientation (ϕ). Because initial ellipticities and orientations vary, so will the finite ellipticities and orientations. However, the finite distributions of *Rf* and ϕ will not be random, but depend on the amount of strain (*Rs*). Take for example a population of objects with *Ri*=2 and variable orientations in figure 3. We can plot the angle ϕ against the ellipticity *Rf* for all objects. Because all objects started with an ellipticity of 2, the data lie on the line *Rf*=2 in the undeformed state (Fig. 3a). After deformation, the distribution changes (Fig. 3b-c). In general points tend to wander towards a lower ϕ and higher *Rf*.

The *Rf*-ϕ curves for different initial ellipticities can be calculated for different finite strains (Fig. 4). Measured data can be plotted on these graphs. One then looks which finite-strain graph fits the data best. The graphs also show a dashed line that encloses 50% of all data points.

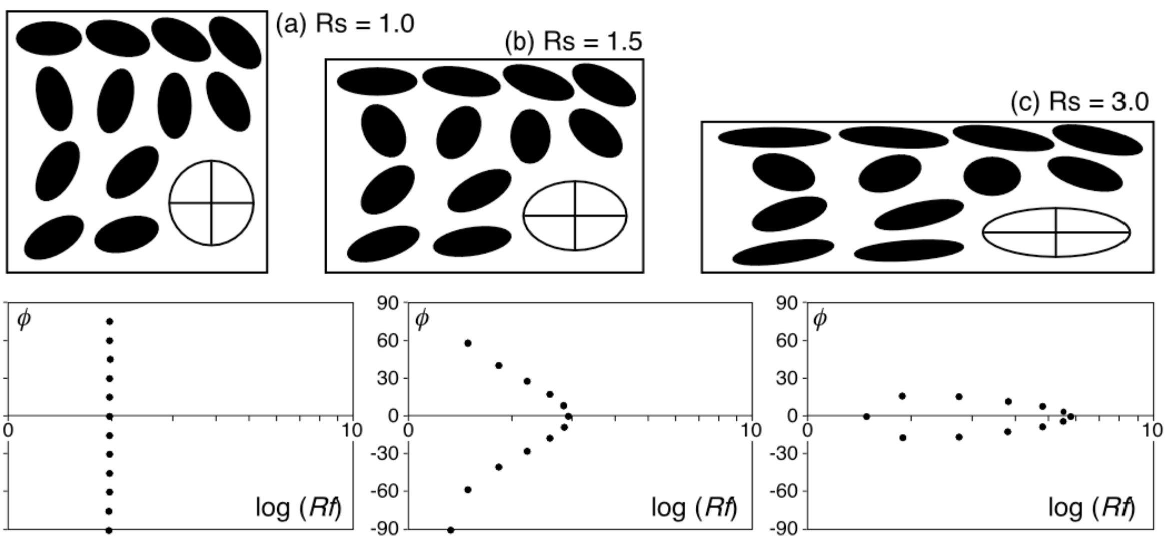
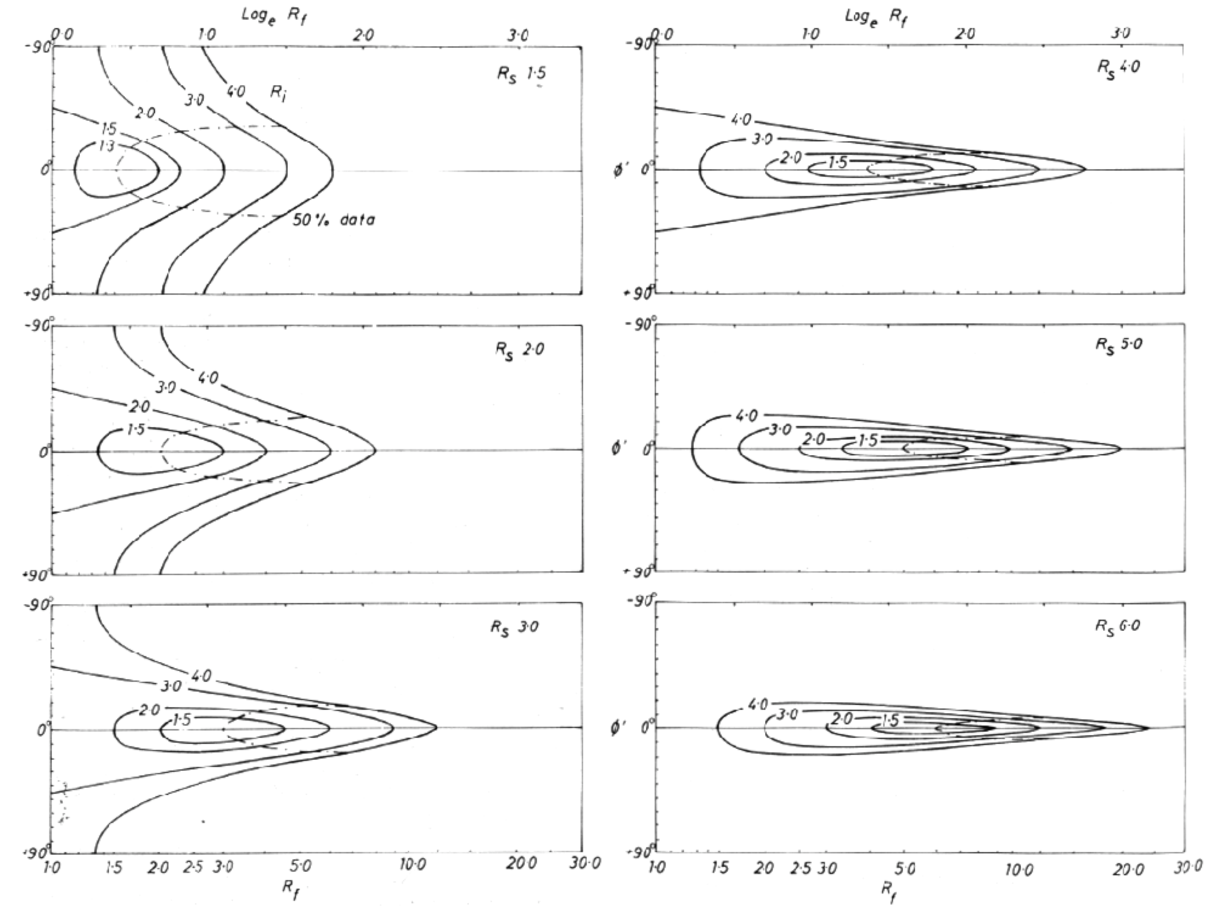
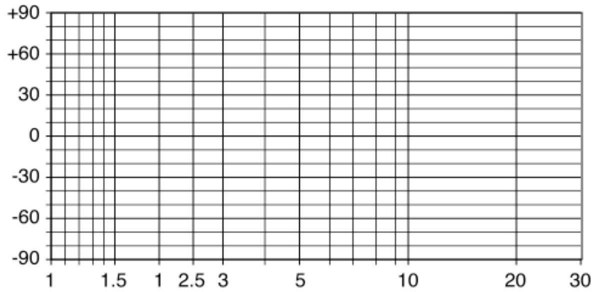


Figure 3. Population of objects with initial ellipticity of *Ri*=2 in the undeformed state **(a)** and after two stages of deformation **(b & c)**. Bottom row: associated *Rf*-ϕ plots.





Plot your data on a sheet of transparent paper using the graph on the left.

Then overlay your plot on the graphs above and estimate which of the Rs values fits your data best

Figure 4. Rf-f graphs for different finite strains.

The Fry method

The Fry method does not directly use the shape of objects, but their spacing. In an isotropic, densely packed aggregate of objects (grains, ooids, etc.), the distances from the centres of objects to those of their neighbours is the same in all directions (Fig. 5a). If such an aggregate is deformed, some of the centres approach each other, while some move apart (Fig. 5b). This is used in the "Fry method".

In the Fry method one plots all the distances between neighbours in a single graph. This done in the following way:

- Take a sheet of transparent paper and put a mark (small cross) in the middle.
- Put the mark on the centre of one object in an image of the deformed rock.
- Mark all the centres of the surrounding grains on the transparency
- Repeat this procedure for all grains in the image.

After doing this, you have a graph that looks like figure 6a. When all the objects originally had an approximately equal distance, a blank ellipse should appear around the central marking: this is the strain ellipse (Fig. 6b).

It should be stressed that this method only works when the object centres were originally equidistant, i.e. roughly the same everywhere. If this is not the case, you will not get a sharply bounded white ellipse and the method does not work.

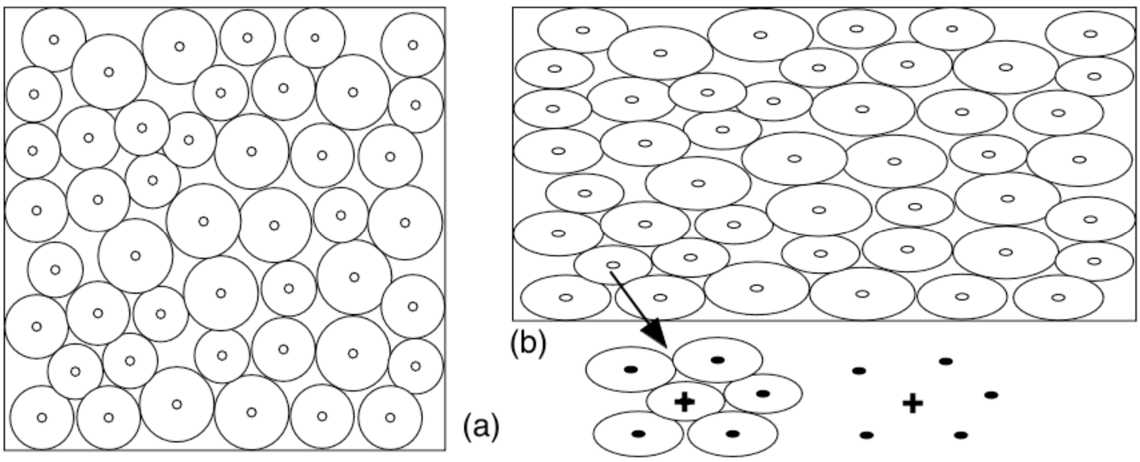


Figure 5. (a) Isotropic aggregate of roughly equal-sized objects (e.g. ooids). **(b)** Same aggregate, but deformed.

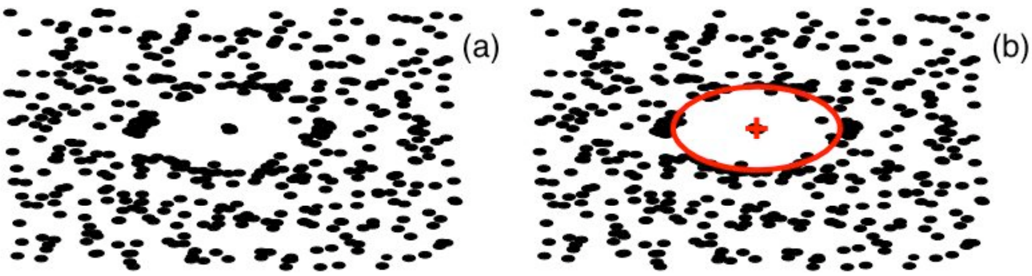
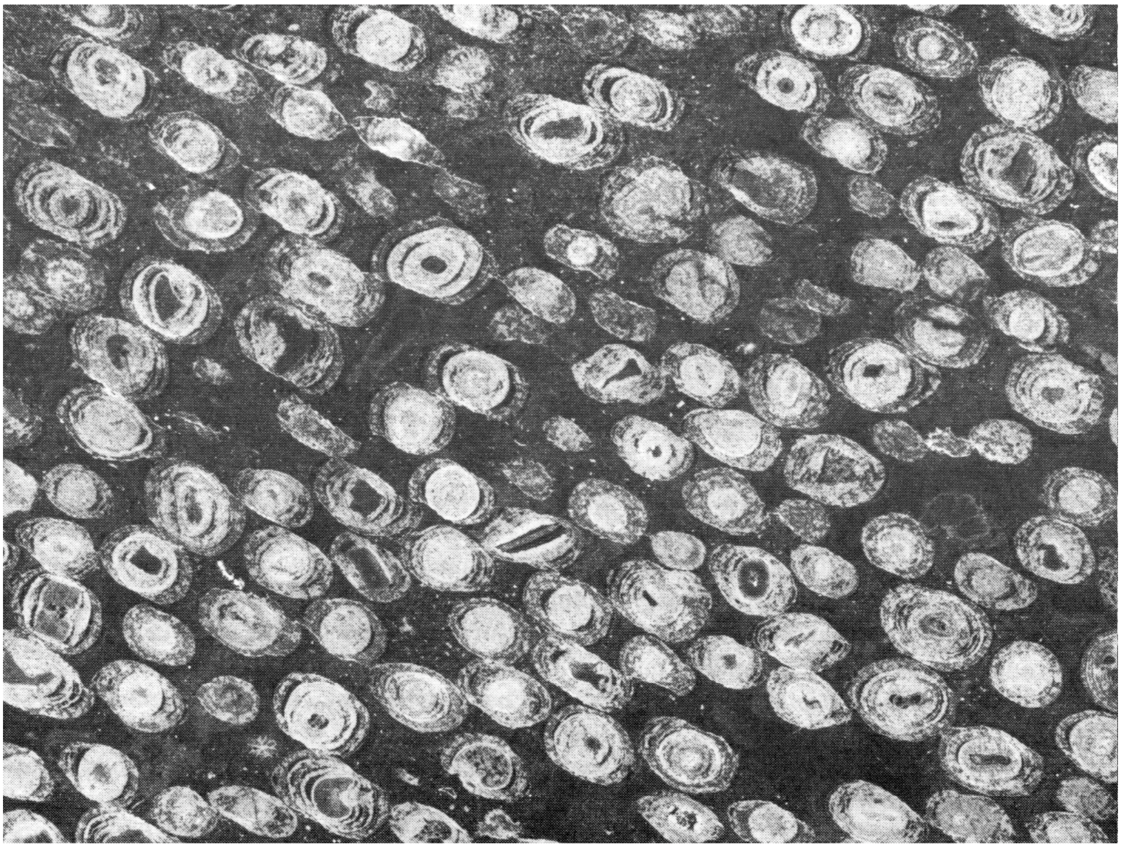
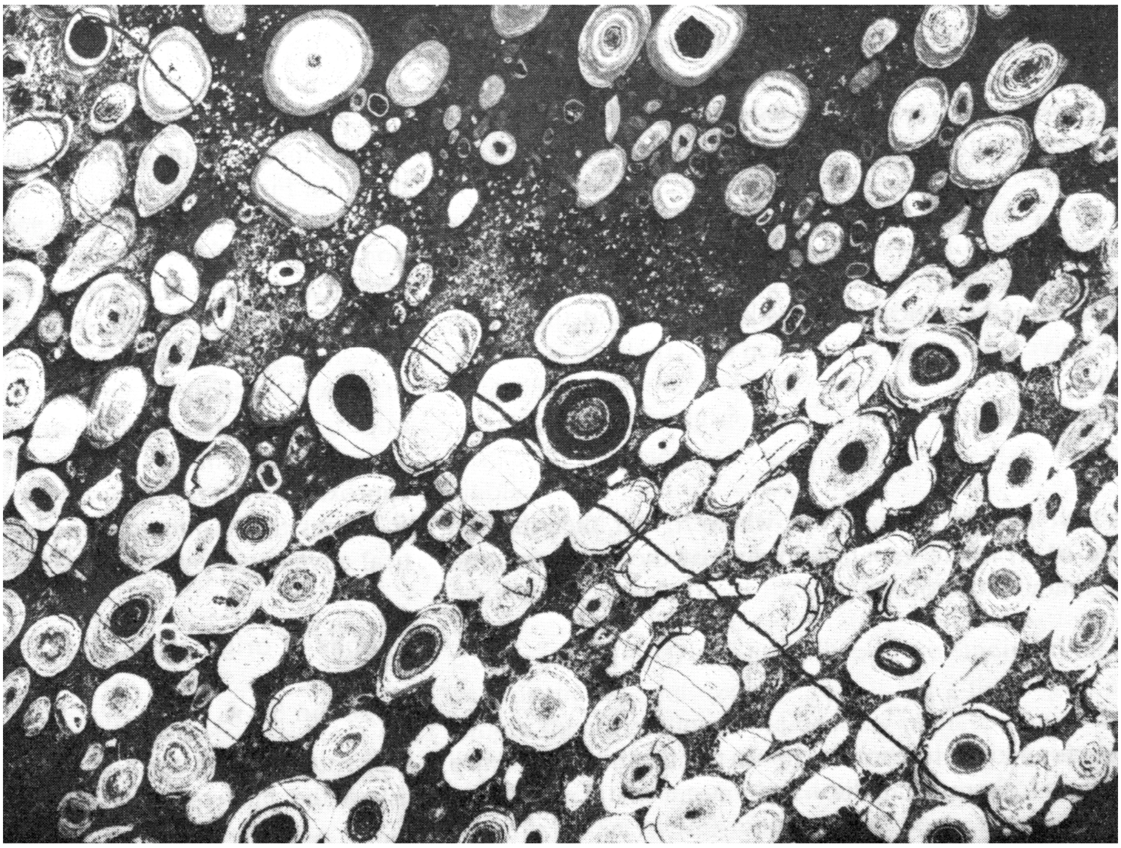


Figure 6. Results of Fry analysis of aggregate of figure 5b. **(a)** Raw result. **(b)** Estimated strain ellipse, which has an axial ratio of 2.

Exercise



Determine strain using Fry method



Determine strain using Rf-ϕ method