Deformation mechanism maps

Minerals deform by different deformation mechanisms, depending on conditions (differential stress, temperature) or state of the material (grain size, strain). A deformation mechanism map shows which mechanism is the dominant deformation mechanism as a function of the variables of interest (e.g. stress-temperature space).

The flow laws for the main deformation mechanism can be expressed as:

$$\dot{\varepsilon} = A \exp^{\left(\frac{-Q}{RT}\right)} \frac{\sigma^n}{g^m}$$

$\dot{\varepsilon}$ = strain rate [s^{-1}]
A = pre-exponential constant [$MPa^{-n}\ m^m\ s^{-1}$]
Q = activation energy [$J\ mol^{-1}$]
R = universal gas constant [$J\ mol^{-1}\ K^{-1}$]
T = temperature [K]
σ = differential stress [MPa]
n = stress exponent []
g = average grain size [m]
m = grain size exponent []

Let us take olivine as an example:

Dislocation creep (grain-size insensitive)

$$\dot{\varepsilon} = 8 \cdot 10^4 \exp^{\left(\frac{-4.5\cdot 10^5}{RT}\right)} \sigma^3$$

$A = 8\cdot 10^4\ MPa^{-3}\ s^{-1}$
$Q = 4.5\cdot 10^5\ J\ mol^{-1}$
$n = 3$
$m = 0$

Nabarro-Herring creep (grain-size sensitive)

$$\dot{\varepsilon} = 2 \cdot 10^{-8} \exp^{\left(\frac{-2.9\cdot 10^5}{RT}\right)} \frac{\sigma}{g^2}$$

$A = 2\cdot 10^{-8}\ MPa^{-n}\ m^m\ s^{-1}$
$Q = 2.9\cdot 10^5\ J\ mol^{-1}$
$n = 1$
$m = 2$

Coble creep (grain-size sensitive)

$$\dot{\varepsilon} = 1 \cdot 10^{-7} \exp^{\left(\frac{-3.5\cdot 10^5}{RT}\right)} \frac{\sigma}{g^3}$$

$A = 1\cdot 10^{-7}\ MPa^{-n}\ m^m\ s^{-1}$
$Q = 3.5\cdot 10^5\ J\ mol^{-1}$
$n = 1$
$m = 3$

Note: these numbers are only approximations for the current exercise

Which mechanism is the most important? That depends on the conditions. Let us consider T = 900°C, σ = 1 MPa and a grain size of 1 mm:

Dislocation creep: $\dot{\varepsilon} = 8 \cdot 10^4 \exp^{\left(\frac{-4.5\cdot 10^5}{R\cdot 1173}\right)} 1^3$ $= 7.3\cdot 10^{-16}\ s^{-1}$

Nabarro-Herring creep: $\dot{\varepsilon} = 2 \cdot 10^{-8} \exp^{\left(\frac{-2.9\cdot 10^5}{R\cdot 1173}\right)} \frac{1}{0.001^2}$ $= 2.4\cdot 10^{-15}\ s^{-1}$

Coble creep: $\dot{\varepsilon} = 1 \cdot 10^{-7} \exp^{\left(\frac{-3.5\cdot 10^5}{R\cdot 1173}\right)} \frac{1}{0.001^3}$ $= 2.6\cdot 10^{-14}\ s^{-1}$

We see that at these conditions, Coble creep is ~10x faster than Nabarro-Herring creep ~35x faster than dislocation creep. The *main deformation mechanism* at these conditions is therefore Coble creep. All other mechanisms are much slower and contribute only a (very) small part of the total strain.

Now consider T = 900°C, σ = 10 MPa and a grain size of 10 mm:

Dislocation creep: $\dot{\varepsilon} = 8 \cdot 10^4 \exp^{\left(\frac{-4.5\cdot10^5}{R\cdot1173}\right)} 10^3$ = 7.3·10^{-13} s^{-1}

Nabarro-Herring creep: $\dot{\varepsilon} = 2 \cdot 10^{-8} \exp^{\left(\frac{-2.9\cdot10^5}{R\cdot1173}\right)} \frac{10}{0.01^2}$ = 2.4·10^{-16} s^{-1}

Coble creep: $\dot{\varepsilon} = 1 \cdot 10^{-7} \exp^{\left(\frac{-3.5\cdot10^5}{R\cdot1173}\right)} \frac{10}{0.01^3}$ = 2.6·10^{-16} s^{-1}

At these conditions, dislocation creep is the dominant mechanism. It is now much faster than both Coble creep and Nabarro-Herring creep.

It would be nice to have a map showing which mechanism is dominant. Each field in the map represents the conditions where a certain mechanism dominates. The boundaries between the fields are the lines where two mechanisms have the same strain rates: they each contribute 50% of the total strain. We will construct such a deformation mechanism map for olivine in stress-temperature space, for a grain size of 10 mm, which is reasonable for the mantle.

1: defining the boundaries between the fields

With 3 deformation mechanisms for olivine, we can define three fields and three boundaries:

$$\dot{\varepsilon}_{Coble} = \dot{\varepsilon}_{Nabarro-Herring}$$

$$\dot{\varepsilon}_{Coble} = \dot{\varepsilon}_{Dislocation}$$

$$\dot{\varepsilon}_{Dislocation} = \dot{\varepsilon}_{Nabarro-Herring}$$

<u>1a: The boundary between Coble and Nabarro-Herring creep</u>

$$\dot{\varepsilon}_{Coble} = \dot{\varepsilon}_{Nabarro-Herring} \Leftrightarrow 1 \cdot 10^{-7} \exp^{\left(\frac{-3.5\cdot10^5}{RT}\right)} \frac{\sigma}{g^3} = 2 \cdot 10^{-8} \exp^{\left(\frac{-2.9\cdot10^5}{RT}\right)} \frac{\sigma}{g^2}$$

$$\Leftrightarrow 1 \cdot 10^{-7} \exp^{\left(\frac{-3.5\cdot10^5}{RT}\right)} \frac{1}{g} = 2 \cdot 10^{-8} \exp^{\left(\frac{-2.9\cdot10^5}{RT}\right)} \quad \Leftrightarrow \frac{\exp^{\left(\frac{-3.5\cdot10^5}{RT}\right)}}{\exp^{\left(\frac{-2.9\cdot10^5}{RT}\right)}} = g \frac{2 \cdot 10^{-8}}{1 \cdot 10^{-7}}$$

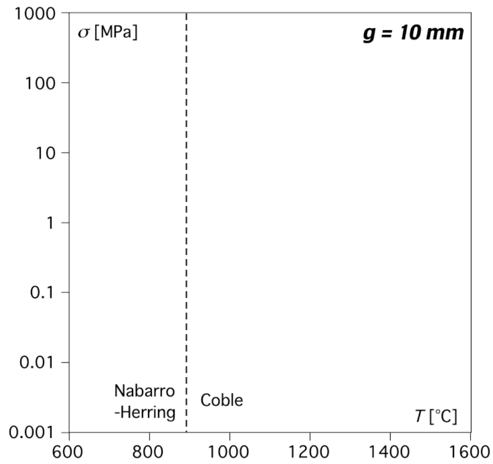
$$\Leftrightarrow \exp^{\left(\frac{-3.5\cdot10^5}{RT} - \frac{-2.9\cdot10^5}{RT}\right)} = \exp^{\frac{1}{RT}\left(-3.5\cdot10^5 + 2.9\cdot10^5\right)} = g \frac{2 \cdot 10^{-8}}{1 \cdot 10^{-7}}$$

$$\Leftrightarrow \frac{1}{RT}\left(-3.5 \cdot 10^5 + 2.9 \cdot 10^5\right) = \ln\left(g \frac{2 \cdot 10^{-8}}{1 \cdot 10^{-7}}\right) \Leftrightarrow T = \frac{\left(-3.5 \cdot 10^5 + 2.9 \cdot 10^5\right)}{R \ln\left(g \frac{2 \cdot 10^{-8}}{1 \cdot 10^{-7}}\right)}$$

If you fill in the grain size of 10 mm, you get that the boundary is at 1161 K (=888°C). Note that the boundary is independent of stress, as both mechanisms have the same stress exponents. The question now arises: is Coble creep dominant *above* or *below* 1161 K? You can find out by filling in a temperature above above 1161 K and a stress, of say 1 MPa (and the grain size of 10 mm, of course).

@ 1200 K and 1 MPa: $\dot{\varepsilon}_{Coble}$ = 3.9·10^{-14} s^{-1}

$\dot{\varepsilon}_{Nabarro-Herring}$ = 1.1·10^{-14} s^{-1}



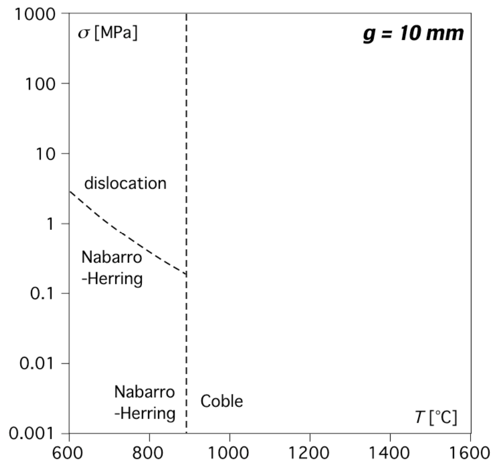
Boundary between Nabarro-Herring and Coble creep

At T>1161 K, Coble creep dominates. This is expected because the activation energy of Coble creep is larger than that of Nabarro-Herring creep. This means that Coble creep is more temperature-sensitive.

1b: The boundary between dislocation creep and Nabarro-Herring creep

$$\dot{\varepsilon}_{Dislocation} = \dot{\varepsilon}_{Nabarro-Herring} \Leftrightarrow 8\cdot 10^4 \exp^{\left(\frac{-4.5\cdot 10^5}{RT}\right)}\sigma^3 = 2\cdot 10^{-8}\exp^{\left(\frac{-2.9\cdot 10^5}{RT}\right)}\frac{\sigma}{g^2}$$

$$\Leftrightarrow \sigma^2 = \frac{2\cdot 10^{-8}}{8\cdot 10^4}\frac{\exp^{\left(\frac{-2.9\cdot 10^5}{RT}\right)}}{\exp^{\left(\frac{-4.5\cdot 10^5}{RT}\right)}}\frac{1}{g^2} \qquad \Leftrightarrow \sigma = \left(\frac{2\cdot 10^{-8}}{8\cdot 10^4}\frac{\exp^{\left(\frac{-2.9\cdot 10^5}{RT}-\frac{-4.5\cdot 10^5}{RT}\right)}}{g^2}\right)^{\frac{1}{2}}$$



The boundary between Nabarro-Herring and dislocation creep

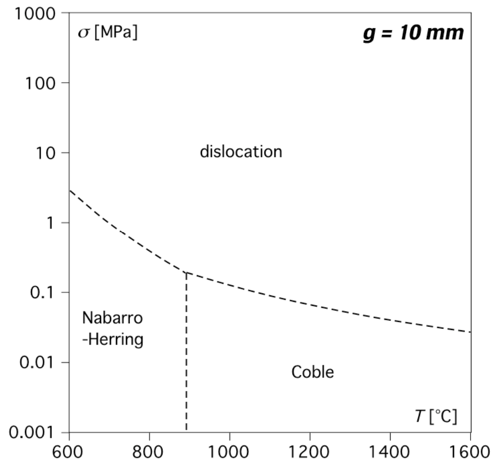
The result is a curve in σ-T space. This line depends on grain size. It should not extend across the Nabarro-Herring to Coble creep boundary.

1c: The boundary between dislocation creep and Coble creep

$$\dot{\varepsilon}_{Dislocation} = \dot{\varepsilon}_{Coble} \Leftrightarrow 8\cdot 10^4 \exp^{\left(\frac{-4.5\cdot 10^5}{RT}\right)}\sigma^3 = 1\cdot 10^{-7}\exp^{\left(\frac{-3.5\cdot 10^5}{RT}\right)}\frac{\sigma}{g^3}$$

$$\Leftrightarrow \sigma^2 = \frac{1\cdot 10^{-7}}{8\cdot 10^4}\frac{\exp^{\left(\frac{-3.5\cdot 10^5}{RT}\right)}}{\exp^{\left(\frac{-4.5\cdot 10^5}{RT}\right)}}\frac{1}{g^3} \Leftrightarrow \sigma = \left(\frac{1\cdot 10^{-7}}{8\cdot 10^4}\frac{\exp^{\left(\frac{-3.5\cdot 10^5}{RT}-\frac{-4.5\cdot 10^5}{RT}\right)}}{g^3}\right)^{\frac{1}{2}}$$

The result is a again a curve in σ-T space. This line depends on grain size.



Map of the mechanism fields

We now have a deformation mechanism map for olivine with a grain size of 10 mm.

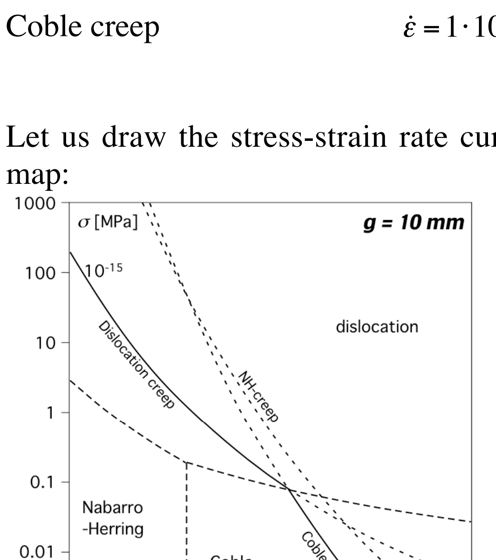
- Nabarro-Herring creep dominates at low stress and low temperature
- Coble creep dominates at low stress and high temperature
- Dislocation creep dominates at high stress

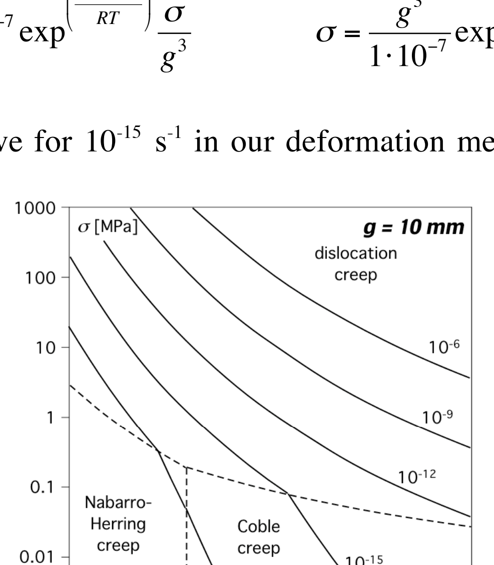
2: Draw the strain rate lines in the map

The deformation mechanism map we now have does not show the strain rates. So, we cannot tell whether the mantle at 1000°C deforms by Coble creep or dislocation creep if the strain rate is 10^{-15} s^{-1}. To find out, we need to know the stress needed to deform at this rate, at 1000°C and a grain size of 10 mm. This stress we can determine from the flow laws, which we just have to invert, so that stress is a function of strain rate:

Mechanism	Strain rate	Stress
Dislocation creep	$\dot{\varepsilon} = 8 \cdot 10^4 \exp^{\left(\frac{-4.5 \cdot 10^5}{RT}\right)} \sigma^3$	$\sigma = \left(\frac{\exp^{\left(\frac{4.5 \cdot 10^5}{RT}\right)}}{8 \cdot 10^4} \dot{\varepsilon} \right)^{\frac{1}{3}}$
Nabarro-Herring creep	$\dot{\varepsilon} = 2 \cdot 10^{-8} \exp^{\left(\frac{-2.9 \cdot 10^5}{RT}\right)} \frac{\sigma}{g^2}$	$\sigma = \frac{g^2}{2 \cdot 10^{-8}} \exp^{\left(\frac{2.9 \cdot 10^5}{RT}\right)} \dot{\varepsilon}$
Coble creep	$\dot{\varepsilon} = 1 \cdot 10^{-7} \exp^{\left(\frac{-3.5 \cdot 10^5}{RT}\right)} \frac{\sigma}{g^3}$	$\sigma = \frac{g^3}{1 \cdot 10^{-7}} \exp^{\left(\frac{3.5 \cdot 10^5}{RT}\right)} \dot{\varepsilon}$

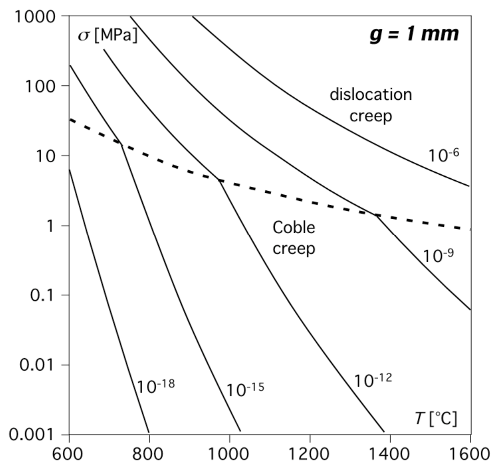
Let us draw the stress-strain rate curve for 10^{-15} s^{-1} in our deformation mechanism map:





We see that to get a strain rate of 10^{-15} s^{-1} at 1000°C and a grain size of 10 mm, we need a stress of about 1 MPa for dislocation creep. For diffusional creep we need a stress about 10x higher. 1 Mpa and 1000°C is in the dislocation creep field, so dislocation creep is the dominant mechanism. Usually curves for a number of different strain rates are drawn in the deformation mechanism map.

One thing we can infer from the map is that it is practically impossible to experimentally study diffusional creep in olivine with a 10 mm grain size. Laboratory strain rates are all well in the dislocation creep field. Does this mean we cannot study diffusional creep in the lab? No, we can! Diffusional creep is very grain-size sensitive, so if we take samples with a small grain size, we expand the fields of Coble and Nabarro-Herring creep. Let us look at the map for a 10x smaller grain size of 1 mm. For dislocation creep this makes no difference, but Nabarro-Herring creep (m=2) would be 10^2 = a hundred times faster and Coble creep even a thousand times. The boundary between Nabarro-Herring and Coble creep now lies at 574°C, so outside the graph we had. The boundary between Coble and dislocation creep has moved up, at the expense of the dislocation creep field.

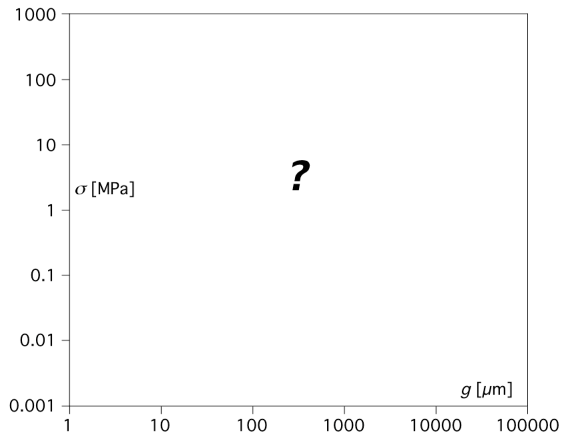


Deformation mechanism maps can be made for any pair of variables. We have already seen a standard combination of stress *vs* temperature (at a certain grain size). One can also make one for stress versus grain size at a certain temperature.

Exercise: Make your own deformation mechanism map

Make a g-σ deformation mechanism map for olivine, using the flow laws given before.

- Plot grain size on the horizontal x-axis, from 1 μm to 10 cm. Use a logarithmic scale.
- Plot stress on the vertical y-axis, from 10^{-3} to 10^{+3} MPa. Use a logarithmic scale.
- The temperature is 800°C.
- Draw constant strain rate curves for 10^{-15} s^{-1}, 10^{-10} s^{-1} and 10^{-5} s^{-1}.



Procedure:

1. Draw the boundaries of the fields:

1a: $$\dot{\varepsilon}_{Coble} = \dot{\varepsilon}_{Nabarro-Herring} \Leftrightarrow T = \frac{\left(-3.5\cdot10^5 + 2.9\cdot10^5\right)}{R\ln\left(g\dfrac{2\cdot10^{-8}}{1\cdot10^{-7}}\right)} \Leftrightarrow g = \frac{1\cdot10^{-7}}{2\cdot10^{-8}}\exp^{\left(\frac{\left(-3.5\cdot10^5+2.9\cdot10^5\right)}{RT}\right)}$$

1b: $$\dot{\varepsilon}_{Dislocation} = \dot{\varepsilon}_{Nabarro-Herring}: \quad \sigma = \left(\frac{2\cdot10^{-8}}{8\cdot10^4}\frac{\exp^{\left(\frac{-2.9\cdot10^5}{RT}-\frac{-4.5\cdot10^5}{RT}\right)}}{g^2}\right)^{\frac{1}{2}}$$

1c: $$\dot{\varepsilon}_{Dislocation} = \dot{\varepsilon}_{Coble}: \quad \sigma = \left(\frac{1\cdot10^{-7}}{8\cdot10^4}\frac{\exp^{\left(\frac{-3.5\cdot10^5}{RT}-\frac{-4.5\cdot10^5}{RT}\right)}}{g^3}\right)^{\frac{1}{2}}$$

2. Draw the stress-grain size curves at the desired strain rates